MY PROBABLE PSEUDO-ELLIPSE IN THE CORRELATIVE DIAGRAM

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INTRODUCTION

I proposed in the former paper "A proposal of organizing an International Association of the Standard Physique" that the authorities in the world who engage in the study of physical status and development, organize the "International Association of the Standard Physique" and determine the International Standard Physique and use it for at least one hundred years in all the countries, and that the authorities in every country organize their branch and determine the Standard Physique of their country or their race and use it for at least ten years. I also made clear that my probable pseudo-ellipse is most suitable for determing the Standard Physique. In this paper I want to demonstrate this theory mathematically and statistically.

I. THE PRINCIPLE OF MY PROBABLE PSEUDO-ELLIPSE

1) Vertical corssing between two curved lines.

The curve of the n-th degree: $y = kx^n$ and the elliptic curve of the n-th kind: $x^2 + ny^2 = r^2$ cross each other perpendicularly at all times in regard to any value of n, k and r.

The proof is as follows:

$$y = kx^n$$

$$\frac{dy}{dx} = knx^{n-1} = \frac{ny}{x} \qquad (1)$$

$$x^2 + ny^2 = r^2$$

$$y = \left\{ \frac{1}{n} (r^2 - x^2) \right\}^{1/2}$$

$$\frac{dy}{dx} = -\frac{x}{n} (r^2 - x^2)^{-1/2} = -\frac{x}{ny} \qquad (2)$$

$$(1) \times (2) = \frac{ny}{x} \times \left(-\frac{x}{ny} \right) = -1$$

Therefore these two curved lines cross each other perpendicularly at all times for any value of n, k and r.

2) The curvilinear co-ordinates.

Regard the group of curved lines crossing mutually and perpendicularly which is made by altering k of $y = kx^n$ and r of $x^2 + ny^2 = r^2$ in proportion to the equal difference. Then k and r become curvilinear co-ordinates, which I call

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the curvilinear co-ordinates of the n-th degree. For example, we use the curvilinear co-ordinates of the height of the body and the girth of chest as that of first degree

$$\begin{cases} B = \frac{b}{100} L \\ L^2 + B^2 = \gamma^2 \end{cases}$$

or the height and weight as that of third degree

$$\begin{cases} W = \left(\frac{F}{1\,000}\right)^3 L^3 \\ L^2 + 3\,W^2 = (G^{3/2})^2 \end{cases}$$

(L denoting the height (cm) and W, the weight (kg), B, the girth of the chest, F, the coefficient of stoutness and thinness, b the coefficient of girth of the chest, and γ and G, the coefficient of γ and G.)

3) *The probable pseudo-ellipse.*

The probable pseudo-ellipse described by the correlation between the height and weight is as follows:

$$\frac{(1\,000\sqrt[3]{W}/L - M_f)^2}{\sigma_f^2} + \frac{(\sqrt{L^2 + 3\,W^2} - M_g)^2}{\sigma_g^2} = X^2$$

(M_f, σ_f, M_g and σ_g denoting respectively the mean value and the standard deviation of the coefficient of stoutness and thinness and of G, and X denoting the coefficient of the inequality of growth in the expression.)

The probable pseudo-ellipse described by the correlation between the height and girth of the chest is as follows:

$$\frac{(100\,B/L - M_b)^2}{\sigma_b^2} + \frac{(\sqrt{L^2 + B^2} - M_r)^2}{\sigma_r^2} = X^2$$

(M_b, σ_b, M_r and σ_r denoting respectively the mean value and standard deviation of the coefficient of girth of chest and that of γ in the expression.)

II. DRAWING METHOD OF MY PROBABLE PSEUDO-ELLIPSE

1) *The correlative diagram.*

We can draw a correlative diagram from two variables of a group: for example, the height-weight correlative diagram is drawn from the height and weight of every man in a group as fig. 1.

2) *Adaptable grade of the curvilinear co-ordinates in the correlative diagram.*

We draw F-G curvilinear co-ordinates in the height-weight correlative diagram, and calculate the number of points in the area bounded by F- and G-curved line as in fig. 2, and then we can calculate the average value (M_f, M_g), standard deviation (σ_f, σ_g) of F- and G-coefficient and the coefficient (R_{fg}) of correlation between F and G in the way heretofore in use. This coefficient (R_{fg}) of correlation between F and G indicates the degree of the adaptation of F-G curvilinear co-ordinates.

In the case of $R_{fg} \leqq 0.1$, the curvilinear co-ordinates are adapted to the correlative diagram.

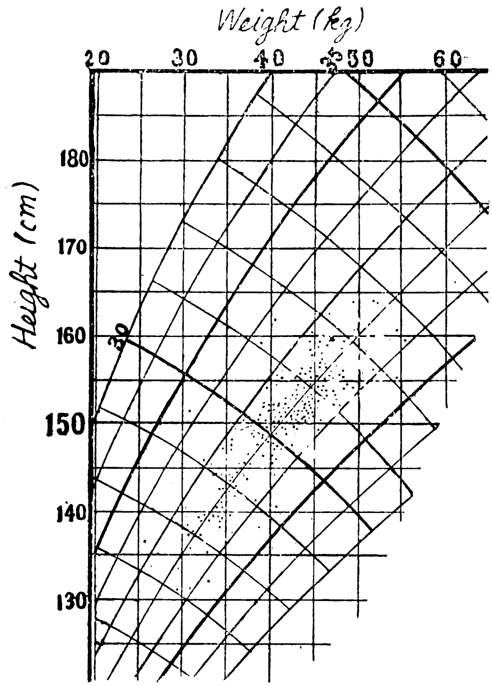

Fig. 1. Correlative diagram between height and weight.

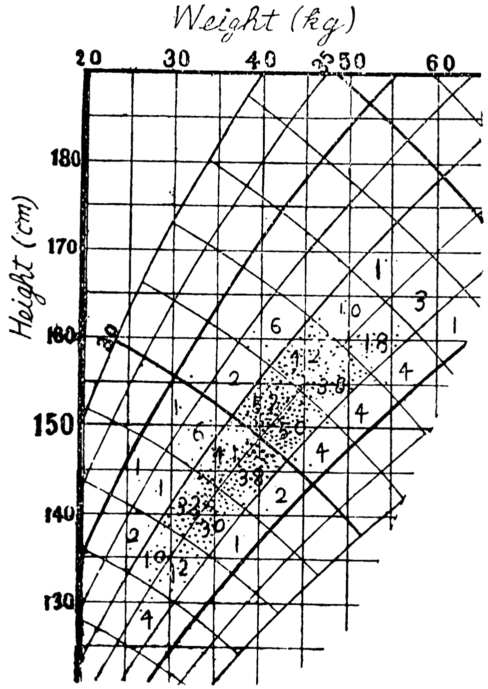

Fig. 2. Correlative table in *F-G* curvilinear co-ordinates.

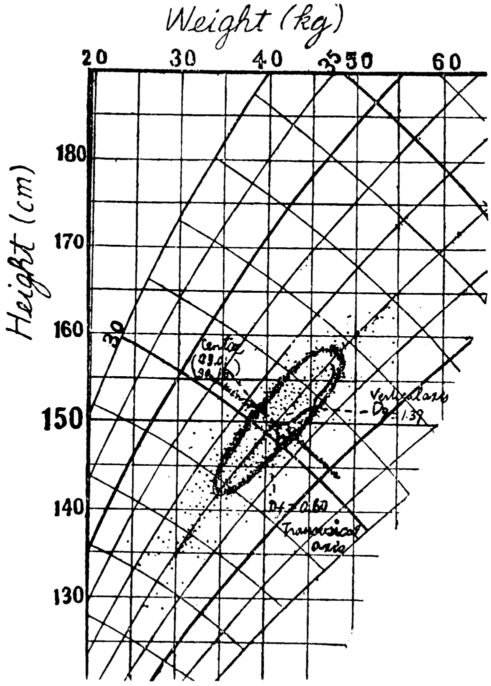

Fig. 3. My probable pseudo-ellipse.

G-coefficient	*F*-coefficient 20	21	22	23	24	25		
							1	
							19	
							64	
34							**101**	
			1	3	1			5
33							102	
			10	18	4			32
32							76	
		6	42	38	4			90
31							42	
		2	52	50	4			**114**
30							14	
	1	6	41	38	2			88
29							5	
	1	1	32	30	2			66
28								
		2	10	12	1			25
27								
				4				4
26								
	2	17	**188**	199	18			424

Calculation of average value and standard deviation and correlative coefficient of F and G

F_f		2	17	188	199	18				$424=N$
E		-2	-1	0	1	2				
F_fE		-4	-17		199	36				$214=\sum F_fE$
F_fE^2		8	17		199	72				$296=\sum F_fE^2$
F_g	4	25	66	88	114	90	32	5		424
E	-4	-3	-2	-1	0	1	2	3		
F_gE	-16	-75	-132	-88		90	64	15		$-142=\sum F_gE$
F_gE^2	64	225	264	88		90	128	45		$904=\sum F_gE^2$
F	5	14	42	76	102	101	64	19	1	424
E^2	25	16	9	4	1	0	1	4	9	
FE^2	125	224	374	304	102		64	76	9	$1282=\sum FE^2$

$A=22.50 \quad L_f=1$ $\qquad$ $B=30.50 \quad L_g=1 \quad N=424$

$m_f=\frac{\sum F_fE}{N}=0.51$ $\qquad$ $m_g=\frac{\sum F_gE}{N}=-0.34$

$S_f=\frac{\sum F_fE^2}{N}=0.698$ $\qquad$ $S_g=\frac{\sum F_gE^2}{N}=2.134 \quad S=3.024$

$M_f=A+m_fL_f=\mathbf{23.01}$ $\qquad$ $M_g=B+m_gL_g=\mathbf{30.16}$

$P_f=\sqrt{S_f-m_f^2-0.083}=0.60$ $\qquad$ $P_g=\sqrt{S_g-m_g^2-0.083}=1.39$

$D_f=P_fL_f=\mathbf{0.60}$ $\qquad$ $D_g=P_gL_g=\mathbf{1.39}$

$m_{Mf}=0.03 \quad m_{Df}=0.02$ $\qquad$ $m_{Mg}=0.07 \quad m_{Dg}=0.05$

$$R_{fg}=\frac{S_f+S_g-S-2\,m_fm_g}{2P_fP_g}=\mathbf{0.09}$$

$R_{fg}=0.09<0.1$ well anpassen.

In the case of $0.4>R_{fg}>0.1$, it is doubtful whether the former is adapted to the latter or not.

In the case of $0.4\leqq R_{fg}$, the former is not adapted to the latter.

In the case of fig. 2, the coefficient R_{fg} is 0.09, so that here the F-G curvilinear co-ordinates are well adapted to the correlative diagram between the height and weight.

3) *Drawing method of the probable pseudo-ellipse.*

We can draw correctly the probable pseudo-ellipse from the average values and standard deviation of F- and G-coefficient by a very complicated method, but practically we can draw it by a very simple method as follows:

The center of this pseudo-ellipse is determined by the average value (M_f, M_g) of F- and G-coefficient and the size of the transversal axis and the vertical axis is determined by the deviation (σ_f, σ_g) of the F- and G-coefficient.

If we want to draw the probable pseudo-ellipse of $X=1$, we designate the center by M_f, M_g and draw the transversal axis which is parallel with the curved line of G-coefficient and the vertical axis which is parallel with the curved line of G-coefficient and then dot at four points which are one D_f and one D_g distant from the center along the transversal and the vertical axis, and then we can draw by guess this probable pseudo-ellipse which passes these four points as in fig. 3.

III. THE COMPARISON BETWEEN MY PROBABLE PSEUDO-ELLIPSE AND THE PROBABLE ELLIPSE HERETOFORE IN USE

Having compared the theoretical value with the observed value as counted by my probable pseudo-ellipse and as by the probable ellipse heretofore in use, I confirmed that my probable pseudo-ellipse is, on the whole, more suitable in each stage of age, as pointed out by the following:

1) The co-ordinate of the centre of every ellipse.

The co-ordinate of the centre of my probable pseudo-ellipse indicates the average value of the *F*- and *G*-coefficients and simultaneously indicates the average value of the height and weight, so that this center corresponds with the center of the probable ellipse heretofore in use.

2) The number of distributed points included in every ellipse.

If the correlative diagram shows the normal correlative curved surface, the number of distributed points included in the probable ellipse can be calculated theoretically as follows:

First, we can derivate the next formula. The probability of the number of the distributed points included in the probable ellipse of X is

$$Px = 1 - e^{-(1/2)X^2}$$

From this formula we can calculate every probability for the case of $X = 1, 2, 3, \ldots .$

Namely:

Table 1

X	$-(1/2)X^2$	$e^{-(1/2)X^2}$	Px
1	−0.5	0.60653	0.39347
2	−2	0.13533	0.86467
3	−4.5	0.01111	0.98889
4	−8	0.0003354	0.9996646
5	−12.5	0.000003726	0.999996274

Comparing this theoretical value with the observed number of distributed points included in the probable ellipse heretofore in use and my probable pseudo-ellipse, the value of my probable pseudo-ellipse (*PPEv*) is more similar to the theoretical value (*Tv*) than that of the probable ellipse (*PEv*) heretofore in use as we see in table 2.

The number of distributed points included in the probable ellipse:

Table 2

Group	N	X	*PPEv*	*Tv*	*PEv*
20 year old men in Gifu Prefecture	34662	1	13656.8	13640	14139.2
		2	29864.7	29975	29834.3
		3	34187.9	34280	34195.4
15 year old boys in the middle school of Tokyo	3703	1	1450.0	1456	1590.8
		2	3198.4	3202	3409.5
		3	3646.8	3662	3652.5

3) The density on the curved line of the probable ellipse heretofore in use and my probable pseudo-ellipse.

If the correlative diagram shows the normal correlative curved surface, the density on every curved line of the probable ellipse can be calculated theoretically as follows: Namely, the density on the curved line of every ellipse is

$$Z=\frac{N}{2\pi\sigma_x\sigma_y\sqrt{1-p^2}}\exp\left[-\frac{1}{e(1-p^2)}\left\{\frac{(x-\xi_1)^2}{\sigma_x^2}-\frac{2p(x-\xi_1)(y-\xi_2)}{\sigma_x\sigma_y}+\frac{(y-\xi_2)^2}{\sigma_y^2}\right\}\right]$$

Comparing this theoretical value with the observed value of density on the curved line of every ellipse, the value of my probable pseudo-ellipse (*PPEv*) is more similar to the theoretical value (*Tv*) than that ot fhe probable ellipse (*PEv*) as we see in table 3.

Table 3. The density on the curved line of every ellipse

Group	*N*	*X*	*PPEv*	*Tv*	*PEv*
20 year old men in Gifu Prefecture	34663	1	580.9	581.2	571.8
		2	127.3	129.7	121.4
		3	11.2	10.7	10.2
15 year boys in the middle schools of Tokyo	3703	1	62.0	62.1	54.8
		2	11.8	13.9	8.9
		3	1.1	1.1	1.0

CONCLUSION

In this paper I introduced my statistical method in curviilinear co-ordinates fashion, explained the darwing method of my probable pseudo-ellipse and demonstrated that my probable pseudo-ellipse is more suitable than the probable ellipse heretofore in use by comparison of the theoretical value with the observed value of the number of distributed points included in every ellipse and the density on the curved line of every ellipse.

From these points of view, I expect that this probable pseudo-ellipse will be popularized and used for determining the International Standard Physique and for comparing the physiques of every country or race and every group.

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