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THE ELASTIC CONSTANT OF TISSUE IN THE BODY ESTIMATED FROM COMPUTERIZED TOMOGRAPHY AND ULTRASONOGRAPHY —THEORETICAL ANALYSIS—

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ABSTRACT

This paper describes a method for calculating the elastic constant of tissue. Length measurements obtained from Ultrasonograms (US) are different from the results obtained from Computerized Tomograms (CT) when the velocity of sound is compared to standardized water measurements. The density of tissue ρ_t can be approximated by the equation, $\rho_t = 1 - N_c/N_{air}$, where N_c and N_{air} are the CT-values of tissue and air respectively. The elastic constant (bulk modulus) K_t, sound velocity V, and density ρ_t are related through the following equation, $V = (K_t/\rho_t)^{0.5}$. The elastic constant K_t is then calculated by, $K_t = K_w (L_c/L_u)^2 \cdot (1 - N_c/N_{air})$, where K_w is the bulk modulus of water, and L_c and L_u are the measured distances from CT and US images respectively.

Keywords: Tissue Charaterization, Tissue Mean Density, Computerized Tomography, Ultrasonography, Tissue Mean Elastic Constant.

INTRODUCTION

Elasticity is defined as the property in a material that allows it to return to its original shape, dimensions, and/or position after an external force distorted or changed its physical integrity. Young's ratio, rigidity, Poisson's ratio, and bulk modulus are frequently used as practical elastic constants[1]. They indicate the force per unit area required to produce unit displacement when external force is applied to objects in various directions. They correspond to expansion or shrinking, shear or tortion, elongation or shrinking of a transverse direction to the force, and to volume change, respectively. If distortion is produced at a point in elastic material, it is transmitted as longitudinal wave or side wave. The longitudinal wave (sound wave) velocity V can be shown as a function of bulk modulus K, and as the density ρ of isotopic material. The equation is written as follows:

$$V = \sqrt{K/\rho} \qquad (1).$$

Therefore, the bulk modulus can be calculated if V and ρ are known. However, the elastic constant of human tissue cannot be theoretically estimated due to its complex structure and composition. The elastic constant of tissue, however, may be a function of the elasticity

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explained above. A basic method for obtaining the elastic constant of tissue by using Computerized Tomograms (CT) and Ultrasonograms (US) is presented in this paper.

TISSUE DENSITY OBTAINED FROM CT-VALUES

When X-rays of an initial intensity of I_0 pass through a tissue of the linear attenuation coefficient of μ_t and of the thickness x, the X-ray intensity of I becomes

$$I = I_0 \exp(-\mu_t x) \qquad (2).$$

The natural logarithm of relative attenuated intensity is

$$\ln(I/I_0) = -\mu_t x \qquad (3).$$

The CT-value N_c on the cross section of the CT corresponds to the value of the left member of the equation (3), and the CT-value indicates the X-ray attenuation of each matrix element (pixel), using water as a standard. This is interpreted as follows:

$$N_c = A(\mu_t - \mu_w) \qquad (4).$$

where μ_w is the linear attenuation coefficient for water, and A is a constant. When the physical density of tissue and water are ρ_t and ρ_w, respectively, the equation (3) becomes

$$N_c = A[(\mu/\rho)_t \rho_t - (\mu/\rho)_w \rho_w] \qquad (5).$$

where $(\mu/\rho)_t$ and $(\mu/\rho)_w$ are the mass attenuation coefficients of the tissue and water, respectively.

Table 1 shows the element composition of water and tissues of the body[2]. The mass attenuation coefficients of these materials were calculated for different photon energies, and tabulated in Table 2. These basic data for mass attenuation coefficients for each element were used by Evans[3]. The X-ray energy presently used for CT scanning is 120–140 keV, and its effective energy range is 50–70 keV. When the mass attenuation coefficient of tissue is compared with that of water for energy above 50 keV, the value is lower than 7% for adipose tissue and larger than 2% for skin. The values for striated muscle, gray matter, and kindeys coincide with that of water within 1%. And, the $\rho_w = 1.0$ g/cm^3 equation (5) becomes

$$N_c = A(\mu/\rho)_w(\rho_t - 1) \qquad (6).$$

N_c is expressed by the linear form of ρ_t. This is also derived from the fact that the electron density, and the number of electrons per gram of tissue is constant for different tissues. Phelps[4] and Parker[5] compared the CT-values N_c, the electron density, the number of electrons per cm^3 for various liquid and plastics, and obtained the following equation:

$$N_c = B(\xi_t - \xi_w) \qquad (7).$$

where B is a constant, and ξ_t and ξ_w are the electron densities of material and of water, respectively. Let the density of the material and water by ρ_t and ρ_w and be shown as follows:

$$N_c = B[(\xi/\rho)_t \rho_t - (\xi/\rho)_w \rho_w] \qquad (8).$$

Table 1. Element Composition (weight percent) and Electron Density (electrons/g)

Element	water	adipose tissue	striated muscle	femur	skin	gray matter	kidney
$_{1}H$	11.2	11.2	10.2	8.4	8.1	9.2	8.8
$_{5}B$					0.009		
$_{6}C$		57.3	12.3	27.6	26.1	16.6	20.7
$_{7}N$		1.1	3.5	2.7	4.43	1.65	2.46
$_{8}O$	88.8	30.3	72.9	41.0	64.4	73.3	69.7
$_{11}Na$			0.085		0.233	0.163	0.165
$_{12}Mg$			0.02	0.2	0.005	0.02	0.021
$_{15}P$			0.175	7.0	0.03	0.225	0.17
$_{16}S$		0.06	0.5	0.2	0.29	0.065	
$_{17}Cl$			0.078		0.283	0.165	0.2
$_{19}K$			0.368		0.064	0.288	0.17
$_{20}Ca$			0.007	14.7	0.011	0.01	0.02
$_{26}Fe$			0.025		0.001	0.03	0.041
$_{29}Cu$			0.001		0.003	0.006	
$_{50}Sn$			0.021		0.003		0.005
$_{82}Pb$					0.08		
Electron density (electrons/g) $\times 10^{23}$	3.34	3.31	3.32	3.19	3.37	3.33	3.34

Table 2. The Mass Attenuation Coefficient (cm^2/g) of Tissues

Energy (keV)	water	adipose tissue	striated muscle	femur	skin	gray matter	kidney
10	5.211	3.208	5.468	20.346	5.098	5.307	5.114
15	1.596	1.051	1.691	6.331	1.644	1.642	1.587
20	0.778	0.552	0.820	2.793	0.830	0.799	0.776
30	0.371	0.303	0.389	0.968	0.391	0.377	0.372
40	0.267	0.237	0.274	0.517	0.277	0.269	0.267
50	0.225	0.210	0.227	0.352	0.230	0.225	0.225
60	0.205	0.196	0.206	0.279	0.210	0.205	0.205
80	0.184	0.180	0.184	0.214	0.187	0.184	0.184
100	0.166	0.164	0.166	0.182	0.174	0.167	0.168
150	0.151	0.150	0.150	0.155	0.154	0.151	0.151

From Evans, R. D. (ref. 3).

where $(\xi/\rho)_t$ and $(\xi/\rho)_w$ are the electron densities per gram for the material and water, respectively. The value of (ξ/ρ) for water is equal to that of soft tissue within 1% as shown Table 1. Equation (8) as $\rho_w = 1.0\ g/cm^3$ becomes

$$N_c = B\,(\xi/\rho)_w\,(\rho_t - 1) \quad (9).$$

This equation is the same as equation (6) and the CT-value is a function of the physical density of material. It is also clear from Phelps's data that the CT-value increases linearly with the electron density of material as well as with the physical density within a 1% error. Using equation (4), the CT-value of air is

$$N_{air} = A\,(\mu_a - \mu_w)$$
$$A\mu_w\,((\mu_a/\mu_w) - 1) \qquad (10).$$

where μ_a is the linear attenuation coefficient for air. When the value of 60 keV of photon energy[3] is used, the ratio of the value of the linear attenuation coefficient for water to that of air is

$$\frac{\mu_a}{\mu_w} = \frac{2.24 \times 10^{-4}\,(cm^{-1})}{2.05 \times 10^{-1}\,(cm^{-1})}$$
$$= 1.09 \times 10^{-3}.$$

The ratio is very small compared with 1.0, and equation (10) becomes

$$N_{air} = -A\mu_w \qquad (11).$$

The linear attenuation coefficient of water μ_w is the same as the mass attenuation coefficient $(\mu/\rho)_w$, respectively:

$$N_{air} = -A\,(\mu/\rho)_w \qquad (12).$$

equation (6) becomes

$$N_c = -N_{air}\,(\rho_t - 1) \qquad (13).$$

The physical density of tissue, therefore, is expressed as:

$$\rho_t = 1 - N_c/N_{air} \qquad (14).$$

and therefore can be estimated from the CT-value of tissue.

THE ELASTIC CONSTANT OF TISSUE AND SOUND VELOCITY

The CT-values of body cross section indicate the distribution of physical density of tissues as shown in equation (13). Images by ultrasound are produced by reflected sound waves, or echoes, which are made at the surface between tissues whose acoustic impedance is different. Acoustic impedance is expressed by the product of the physical density of tissue and the velocity of sound in tissue. The velocity of sound is determined by the elastic constant and the density of the material as shown in equation (1). If the velocity of sound in the tissue differs from that in water, the US image of tissue will be distorted and will differ from the CT image, as the CT image shows a non-distorted cross section of the body. The thickness estimated from the CT image $L_{c(t)}$ is the actual length L, but the thickness estimated from the US image $L_{u(t)}$ cannot be the same as L even if the scale on the US image is corrected with a water phantom. If the US image of the tissue is smaller than the CT image of the same cross section, the reduction rate r is

$$r = L_{u(t)}/L_{c(t)} \qquad (15).$$

When a water phantom whose thickness is L shows the thickness of $L_{u(w)}$ on the US image, the following equation will be established:

$$L_{u(w)} = L_{c(w)} = L_{c(t)} \qquad (= L) \qquad (16).$$

The thickness (or length) on the US image increases or decreases with the velocity of sound, and

$$r = V_w/V_t \qquad (16)$$

where V_w and V_t are the velocities of sound in water and in the tissue, respectively. By inserting equation (1):

$$r = ((K_w\rho_t)/(K_t\rho_w))^{0.5} \qquad (18)$$

where K_w, ρ_w, and K_t, ρ_t are the elastic constants and physical densities for water and the tissue. When $\rho_w = 1.0\ g/cm^3$, equation (18) becomes

$$K_t = K_w\rho_t/r^2 \qquad (19).$$

By inserting equation (14) and (16),

$$K_t = K_w \frac{L_c^2}{L_u^2}\left(1 - \frac{N_c}{N_{air}}\right) \qquad (20)$$

The elastic constant of the tissue, therefore, can be estimated from the CT-value and the reduction rate of the US image.

DISCUSSION

When X-rays pass through material, they are absorbed and/or scattered by the photoelectric and Compton effects. The mass attenuation coefficients of the photoelectric effect is 1/10 of the Compton effect at the photon energy of 50 keV. Whereas for bone tissue, the mass attenuation coefficients of both effects are the same at 40 keV, and the mass attenuation coefficient of the photoelectric effect is 1/10 of that of the Compton effect at 100 keV[6]. The Compton effect compared with the photoelectric effect occurs much more frequently in soft tissue in the energy range of X-rays used for CT scanning. The photoelectric effect, however, should not be ignored in bone tissue. Attenuation by the Compton effect increases with the electron density, (the number of electrons per gram), of material. X-ray photons of low energy attenuate to a greater degree than those of high energy due to their broad energy spectrum. The above phenomenon is called "beam hardening" because the effective energy becomes higher than the original energy used. As the attenuation of X-rays in bone tissue is large, the effect of beam hardening for CT scanning of cranial tissue is large[7], and is also recognized in the abdomen where bone tissue is minimal. Herman[8] reported changes in the CT values of the abdomen by computer simulation, and he estimated a 6% change in CT-values. The changes in the CT-values decreased by 3% when the CT-values were corrected with an abdominal sized water phantom.

Attenuation absorption, reflection, and velocity of sound in tissue, in spite of intensive investigations, is difficult to measure in the human body in vivo due to the complex structure of human tissue. The exception to this is the measurement of sound velocity.

The velocity, density, and elastic constant can now be measured or calculated in vivo by the method presented in this paper. Other acoustic properties, *i.e.*, attenuation ratio, and the reflection ratio can be easily determined on the bases of these values.

The reliability of this method depends on the CT-value and the spatial resolutions of CT and US equipment. It is necessary to determine the elastic constant error, and develop a clinically practical processing system for the application of this method.

CONCLUSION

The elastic constant of tissue can be obtained by using in vivo Computerized Tomograms and Ultrasonograms as a function of CT-values, the lengths measured from CT, and US images. The elastic constant K_t can be derived from following equation: $K_t = (L_c^2/L_u^2) \cdot (1 - N_c/N_{air})$, where K_w is the bulk modulus of water, L_c, L_u, N_c, and N_{air} are the measured distances from CT and US image, and CT-value of tissue and air respectively.

REFERENCES

1) Slater, J. C. and Frank, N. H.: Introduction to Theoretical Physics. McGrow-Hill Book Co., Inc., New York, 1933.
2) Sakuma S.: Fukubu X-sen Kaibogaku, pp. 221—229, Kanehara Shuppan, Tokyo, 1977 (in Japanese).
3) Evans, R. D.: X-ray and γ-ray interactions. (in) Radiation Dosimetry, 2nd ed. Vol. 1, pp. 112—144, ed. by H. Attix and W. C. Roesch, Academic Press, New York, 1968.
4) Phelps, M. E., Gado, M. H., and Hoffman, E. J.: Correlation of effective atomic number and electron density with attenuation coefficients measured with polychromatic X-rays. *Radiology,* **117**, 585—588, 1975.
5) Parker, R. P., Hobday, P. A., and Cassell, K. J.: The direct use of CT numbers in radiotherapy dosage calculations for inhomogeneous media. *Phys. Med. Biol.*, **24**, 802—809, 1979.
6) Ter-Pogossian, M. M.: The Physical Aspects of Diagonostic Radiology. pp. 161—174, Harper & Row Pub., New York, 1967.
7) Nalcioglu, O. and Lou, R. Y.: Post-reconstraction method for beam hardening in computerized tomography. *Phys. Med. Biol.*, **24**, 330—340, 1979.
8) Herman, G. T.: Correction for beam hardening in computed tomography. *Phys. Med. Boil.*, **24**, 81—106, 1979.
9) Chivers, R. E.: Tissue Characterization. *Ultrasound in Med. & Biol.*, **7**, 1—20, 1981.